Re-Exam Mathematical Physics, Prof. G. Palasantzas

- Date 08-07-2016
- Total number of points 100
- 10 points free for coming to the re-exam
- For all problems justify your answers

Problem 1 (15 points)

Find the value of c for which $\sum_{n=0}^{+\infty} e^{nc} = D\,(>1)$

Problem 2 (15 points)

Consider the power series: $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^{3/2}} x^n$

For which values of x is the series convergent?

Problem 3 (10 points)

Determine the Taylor series of $f(x) = A/(x-B)$ at the point x=c (≠B) using geometric series and give its interval of convergence.

Problem 4 (15 points)

If a, b, and c are all positive constants and $y(x)$ is a solution of the differential equation $ay'' + by' + cy = 0$, show that $\lim_{x\to\infty} y(x) = 0$.

Problem 5 (15 points)

Find the periodic solutions in complex form of the equation $\frac{d^m W}{dx^m} + AW = f(x)$

with A (≠0) a real (non-integer) number, and f (x) a known $2L$-periodic function.

Consider the Fourier series for f and W: $f(x) = \sum_{n=-\infty}^{n=+\infty} f_n e^{i(n\pi x/L)}$, $w(x) = \sum_{n=-\infty}^{n=+\infty} y_n e^{i(n\pi x/L)}$

Problem 6 (20 points)

Assume a function *f(x)* has the Fourier transform: $F(k) = \int_{-\infty}^{+\infty} f(x) e^{-i2\pi kx} dx$

Consider the definition of the Dirac delta function: $\delta(k) = \int_{-\infty}^{+\infty} e^{-i2\pi kx} dx$

(a: 10 points) Derive the Fourier transform of the function:

$$f(x) = cos^2(2\pi k_o x)$$

(b: 10 points) Derive the Fourier Transform of the function:

$$f(x) = cos^3(2\pi k_o x)$$

Tip: cos(x)=(e^{ix} +e^{-ix})/2